Evaluation of Piled Raft Behavior based on the Taguchi Method subjected to Combination of Vertical and Horizontal Loads

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Abstract

The results of an ongoing FEM parametric study are presented regarding the dependence of the resulting piled raft behavior under lateral load and combination of loads on pile diameter, pile length, arrangement of piles and raft thickness. Taguchi method with Analysis of Variance (ANOVA) was employed to calculate the contribution ratio of these factors on the lateral displacement of piled raft. The obtained results of this study show that the pile diameter is an effective factor in horizontal deformation of the piled raft under pure horizontal load. However, in the case of load combinations, the pile length has the highest participation ratio in reducing the horizontal deformations.

Keywords: Piled raft, Optimum design, Vertical load, Lateral load, Taguchi method.

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Introduction

Piled rafts are usually subjected to vertical loads transferred from overburden pressure and lateral loads from winds, waves, earth pressure or earthquake. Therefore, the analysis of this type of foundation under lateral loads is crucial. Analyzing the effects of different factors on horizontal deformation of piled raft can be a new step in the study of piled rafts behavior under load combination.

Piled raft foundations are often used to reduce the foundation settlement to an acceptable level. There are numerous mechanical properties that have effect on total and differential settlement of piled raft. Poulos [1] suggested that increasing the cap thickness can reduce differential settlement significantly. In addition, he concluded that increasing the number of piles is beneficial generally; however, it does not cause to the best performance of foundation.

Differential settlement of the piled raft depends on the type of loading (i.e. uniform load, line loads, and point loads) [2]. As a practical case, the Messeturm tower in Frankfurt was designed using piled raft foundation in which more piles were arranged closer together on the edges [3]. Base on this study it can be deduced that the piles arrangement and the relative stiffness of the raft-soil system have high impact in reducing the total and differential settlement. Nguyen et al. [4] investigated the effects of some factor using the centrifuge test. In their study they evaluated concentrated and uniform arrangement and have shown that total and differential settlements are reduced by centralizing the piles around the point load (i.e. where the columns are placed) and thereby the design becomes more economical and optimal.

Brown et al. [5] showed that the behavior of piles depends on their position in the group and the ultimate lateral resistance within a pile group is reduced because of the overlapping of the stress zones in the surrounding soil. The lateral resistance of pile group is a function of the row position in the group. Also, Rollins et al. [6] concluded that for a given displacement, the piles in the trailing row tolerated less load compared to the piles in the leading row and all the piles in group tolerated less load in comparison to a single pile due to the effect of group. Comodromos and Pitilakis [7] and Kim and Yoon [8] have shown in their studies that the average lateral resistance is a function of the pile spacing and that the group interaction has substantial effect on reducing pile lateral resistance. The pile behavior under combination of loads has been investigated in a few experimental studies. Meyerhof et al. [9] studied the effect of lateral load and bending moment on the bearing capacity of single pile and pile group. They observed considerable reduction in vertical bearing capacity if there are lateral loads or bending moment. On the other hand, regarding the effect of the vertical loads, the available results in the technical literature are in contrast with each other. Initial analytical investigations by Davisson and Robinson [10] and Goryunov [11] indicate that the lateral displacement of the piled raft foundation under combination of loads is

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increased. Nonetheless, recent studies based on experimental investigation by Zheng et al. [12] and the numerical analyses by Hussien et al. [13, 14] show that the lateral displacement of piled raft foundation under the load combination is reduced. They related this improvement in the lateral capacity to the extra lateral stress of the soil in front of the pile and the developed frictional resistance throughout length of the pile.

As mentioned above, there are different factors involved in the design of the piled raft foundation. In this paper, the Taguchi method was used to quantitatively study the effect of different factors including the pile diameter, pile length, arrangement of piles and raft thickness on the piled raft behavior. Yazdani et al. [15] applied the Taguchi method in the numerical analyses of pile load testing. This method uses orthogonal arrays to organize the factors affecting on the piled raft behavior. Furthermore, the Analysis of Variance (ANOVA) was used to analyze the data obtained from Taguchi method.

Validation of the Finite Element Model

Characteristics of the experimental model

Nguyen et al [4] conducted the centrifuge test using acceleration equal to 50 g. The model used in the finite element analysis consists of a reinforced concrete cap with a dimension of 19×19 m and thickness of 0.542 m. The number of piles is equal to 16 with a diameter of 0.6 m and length of 12.5 m. The piles are uniformly placed at a distance of 5 meters from each other. The vertical loads were applied using 4 columns as shown in Figures 1a and 1b. The total vertical load in the prototype scale is equal to 29000 kN, and the load ratio for columns C1, C2, C3 and C4 is equal to 4, 2, 2 and 1, respectively.

In this study, behavior of the piled raft was investigated by using the finite element code of the Plaxis 3D Foundation. The geometry of the model was divided into 15-node wedge element and a fine mesh assigned to the model. Figure 1c shows 3D finite element mesh of the model. Standard fixities used for boundary conditions. Characteristics of the pile and raft material which is made of concrete listed in Table 1. The piles and raft modeled using the pile element and plate element, respectively. The hardening soil model behavior was selected for loose silica sand.

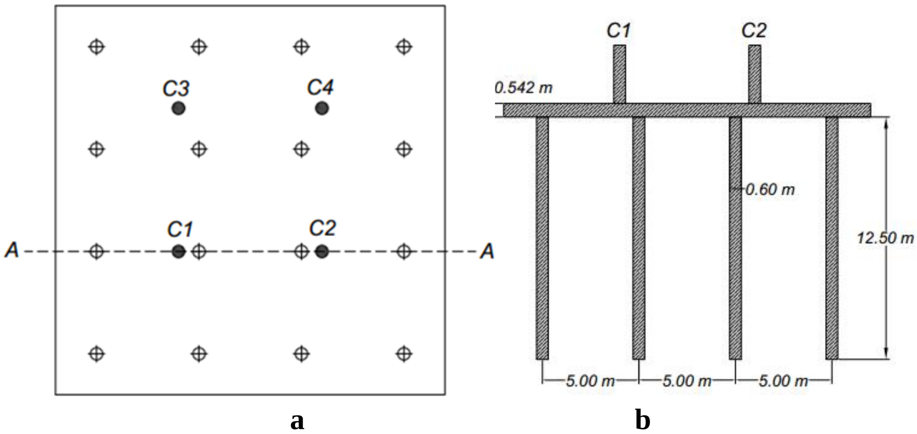


Figure 1. Details of piled raft model; a) Plan section of pile and column arrangement, b) Cross section of piled raft, section A-A,

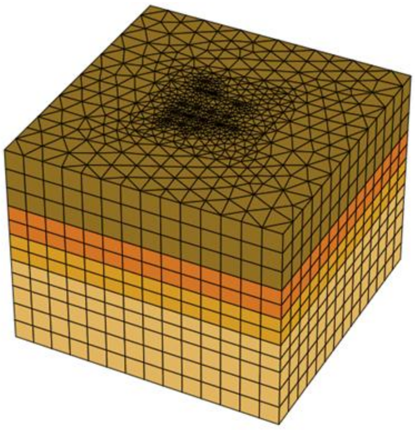


Figure 1. c) Finite element model

Table 1. Pile, raft, and silica sand parameters (Nguyen, 2013)

Parameter	Input value
Soil material	
Dry density, γ_d (kN/m^3)	13.7
Secant Young's modulus, E50 (MPa)	16.11
Friction angle, Ø (degrees)	40
Confinement pressure (kPa)	111.84
Poisson's ratio, ν	0.25
Stress dependent stiffness according to power low, m	0.65
Dilatancy angle, ψ (degrees)	8
Pile and raft material	
Young's modulus, E (MPa)	2.82×10^4
Density, γ (kN/m^3)	15
Possion's ratio, ν	0.16

Simulation of loose silica sand using the hardening soil model

The hardening soil model is an efficient and robust model in simulating soil behavior. This model is thoroughly explained by Brinkgreve [16]. Equation (1) is one of the basic equations in the hardening soil model that shows the relation between stresses and soil stiffness.

$$E_{oed} = E_{oed}^{ref}\left(\sigma / p^{ref}\right)^{m} \quad (1)$$

where, E_{oed}^{ref} is the reference elasticity modulus corresponding to the reference confining pressure; P^{ref} is the relation between the stress and stiffness that this relation is given by the parameter of 'm' with a value between 0.5 and 1.

The material properties of the soil were adopted from triaxial test conducted by Nguyen et al [17], as listed in Table 2. Figure 2 shows the variation of Young's modulus of loose silica sand against the depth. In this figure, solid line represented the experimental Young's modulus and dashed line was given by Equation (1), so that the reference point was chosen at the depth of two-thirds of the pile length from the pile head. As shown in Figure 2, the value of elasticity modulus around the pile toe has a good corresponding with the real value, but the soil stiffness at the initial depth show high values, especially when the pile length is increased and reference point is chosen from the lower depth. This issue creates problems in the design. For example, when a pile is subjected to the lateral load, elasticity modulus around the pile head plays an important role. In fact, by increasing the pile length, the soil model gets away from its real behavior and behaves as a stiff material.

Table 2. Experimental data for loose silica sand (Nguyen 2014)

Depth (m)	Confinement pressure (kPa)	E (MPa)
3.8	50	8.47
7.6	100	13.33
15.2	200	36.84
22.8	300	75.25
34.2	450	112.53

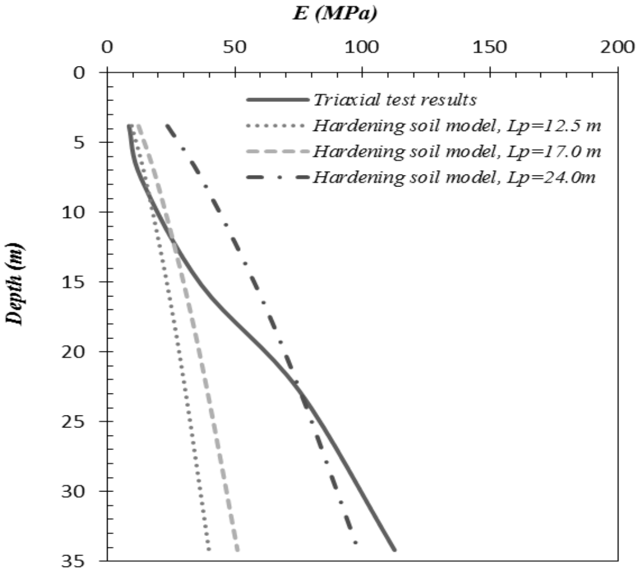


Figure 2. Experimental elasticity modulus and calculated by means of Eq. (1) for different pile length

In the present paper, the soil profile divided into several different layers to solve the aforementioned problem. The hardening soil model selected for soil material modeling and value of soil stiffness assigned as shown in Figure 3. In this case, the elasticity modulus calculated by Equation (1) is closer to the experimental values and consequently the soil behavior under both of the horizontal and vertical loadings are closer to the real value. Also, to comparative study the effect of pile length on the settlement, soil profile remained constant.

Comparing the results of the calculated and measured settlement

The results of numerical simulation using Plaxis 3D Foundation in the A-A section of Figure 1, are shown in Figure 4. As can be seen,

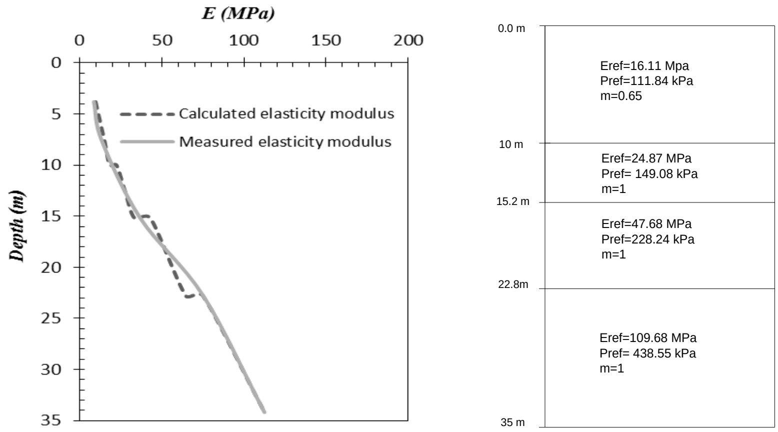


Figure 3. Fitting modulus of elasticity by Eq (1) in layered soil

there is a 30% difference between the result of experimental tests and numerical simulation. Due to not taking into account the group effects in the plaxis 3D Foundation, the settlement obtained from Plaxis 3D Foundation are bigger than the measured settlement in the centrifuge test and that the value of calculated settlement under the C1 column is much higher than the measured settlement [4].

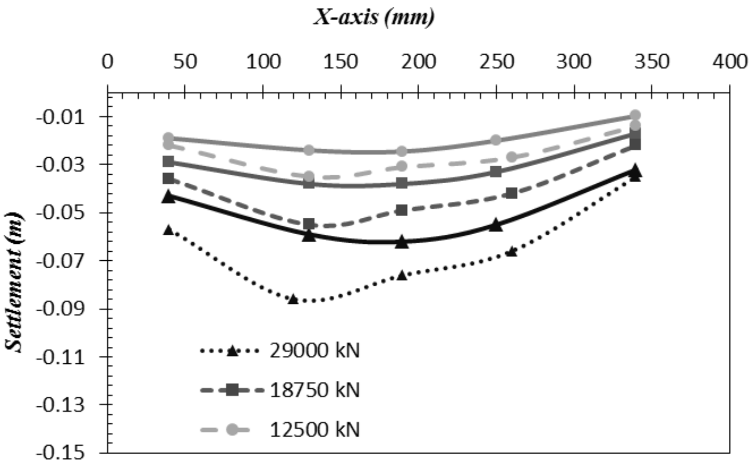


Figure 4. Comparison of the settlement between centrifuge test and numerical simulation, uniform arrangement

Numerical modeling under combination of load

The numerical study of the piled raft was conducted under lateral load and combination of loads considering different pile lengths, pile diameters, arrangements, and raft thicknesses as summarized in Table 3. Arrangements of U, A1, and A2 were used with spacing of 5m, 2.4 m, and 3.5 m, as shown in Figure 5. A square raft with a width of 19 m was considered. The vertical load of 29000 kN was applied to the four columns. The load ratio for columns of C1, C2, C3 and C4 is equal to 4, 2, 2 and 1, respectively. The horizontal load is equal to 40% of the total vertical load. The amount of horizontal load for each column was 2900 kN.

Table 3. Different parameters for evaluating the behavior of piled raft foundation

Arrangement	Pile length(m)	Pile diameter(m)	Raft thickness(m)
U	12.5	0.6	0.5
A1	17.0	0.8	1.0
A2	25.0	1.0	1.5

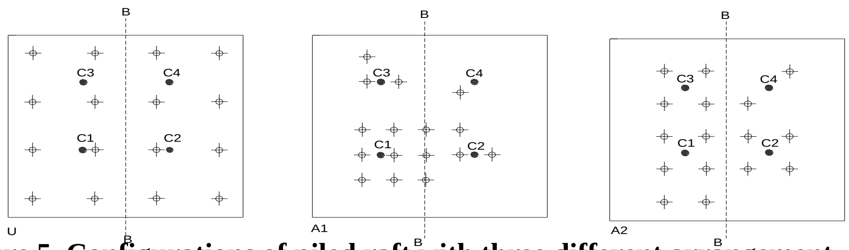


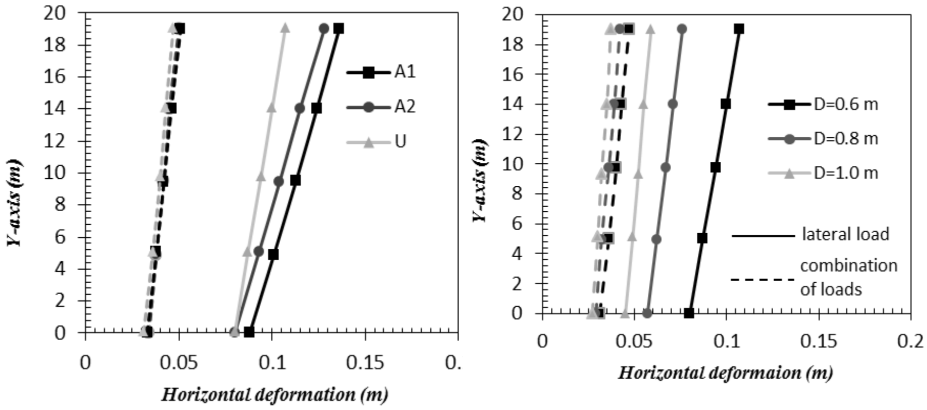
Figure 5. Configurations of piled raft with three different arrangement

Effect of some parameters on the piled raft behavior using Plaxis 3D foundation has been evaluated as depicted in Figure 6. The variations of horizontal deformation against different pile arrangements are presented in Figure 6a. It is worthy to note that the horizontal deformation decreases with increase in pile spacing due to the overlapping the stress bulbs around piles which leading to decrease in the passive resistance. As shown in Figure 6a, the effect of pile spacing becomes less significant when the piled raft is subjected to the combination of loads. Since the vertical loads creates a relative displacement between pile, cap and soil, the horizontal stresses increase around the pile and leading to increase in the resistance of the piled raft against lateral load.

Figure 6b shows a comparison of the horizontal displacement of piled raft with three different pile diameters. The results indicate that the lateral resistance increases with an increase in the pile diameter. Also, increasing the pile diameter can result in increase in the pile stiffness and consequently, horizontal deformation is reduced.

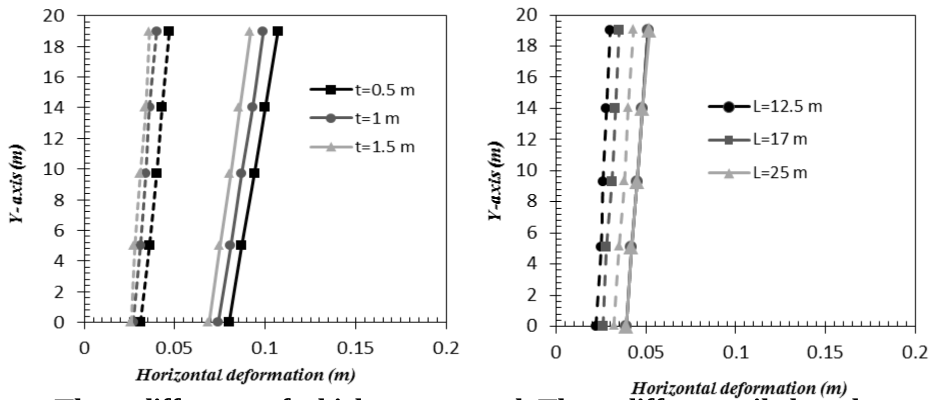
The effect of raft thickness on the lateral deformation is illustrated in Figure 6c. As can be seen in Figure 6c, the flexible raft with a thickness of 0.5 m has the most deformation. But the variations in the deformation between the two rigid rafts with thicknesses of 1.0 m and 1.5 m are minimized.

In this section, it is focused on the effects of the pile length on horizontal displacement of the piled raft with uniform arrangement. Figure 6d shows comparing the deformation of the piled raft with lengths of 12.5 m, 17.0 m, 25 m. As expected, by changing the pile length, the horizontal deformations under lateral loads remained constant. However, the pile length plays an important role in the lateral response of the piled raft due to applying the vertical load before horizontal load.



a. Three different arrangements **b. Three different diameters**

Lp=12.5 m, D=0.6 m, t=0.5 m **U, Lp=12.5m, t=0.5m**



c. Three different raft thicknesses **d. Three different pile lengths**

U, Lp=12.5m, D=0.6 m **U, D=0.6 m, t=0.5m**

Figure 6. Comparing the horizontal deformation of piled raft for different parameters

Figure 7 illustrates the horizontal soil stress in the initial depths to better understand of this issue. It can be seen in Figure 7 that the horizontal soil stress increases with the application of the vertical load and the less pile length causes to the more lateral stress. The reason is that decreasing the pile length leads to the increase in settlement and consequently, the lateral resistance of the piled raft is increased due to increasing the lateral stress around the pile. Therefore, reducing the pile length can lead to the desirable results when the horizontal deformation of the structure is very important, however, the acceptable level of structure settlement should be taking into account in choosing process of the pile length.

Quantitatively study of piled raft based on Taguchi method and ANOVA methodology

Traditional experimental design procedures are too complicated and not easy to use. A large number of experimental works must be carried out when the number of parameters increases. The Taguchi method can be used to solve this problem. This method uses a special set of arrays called orthogonal arrays. These standard arrays stipulate the way

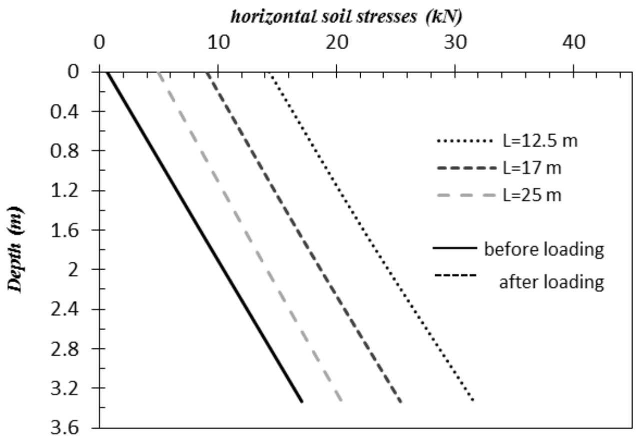


Figure 7. Distribution of horizontal soil stresses along the upper part of pile, before and after loading

of conducting the minimum number of experiments which could give the full information of factors affecting on the performance. In this research four factors including the arrangement, the raft thickness, the diameter and the length of pile, were considered to the design of piled raft. The L9 orthogonal array was used in which there is nine rows corresponding to the number of analysis, as listed in Table 4. Three different arrangements of the piled rafts were considered, as shown in Figure 5. The pile spacing of the U, A1 and A2 arrangements are equal to 5 m, 2.4m and 3.5 m, respectively.

Table 4. The L_9 orthogonal array for evaluating the piled raft behavior

No. of combination	Arrangement	Diameter (m)	Length(m)	Raft thickness (m)
1	A1	0.6	12.5	0.5
2	A1	0.8	17	1
3	A1	1	25	1.5
4	A2	1	17	0.5
5	A2	0.6	25	1
6	A2	0.8	12.5	1.5
7	U	0.8	25	0.5
8	U	1	12.5	1
9	U	0.6	17	1.5

In this section, we study the effect of four different factors on differential settlement, maximum settlement and horizontal deformation of piled raft under vertical load and combination of loads. As mentioned before, nine analyses are enough to understand the effects of the four different factors on differential and maximum settlement and horizontal deformation. Table 5 shows the results of the analysis for the L9 orthogonal array.

Analysis of Variance (ANOVA) was chosen to compare the effect of factors on the settlement and lateral deformation and obtaining the contribution percent of factors including arrangement, raft thickness, Diameter and length of pile. The obtained results help to determine which factors need control.

The contribution percent of factors can be defined as [18]:

$$P_A = \frac{S'_A}{S_T} \tag{2}$$

where P_A is the contribution percent of factors, S'_A is the pure sum

Table 5. The values of settlement and horizontal deformation for each combinations of the L_9 Array

No. of combination	Differential settlement (m)		Maximum settlement (m)		Horizontal deformation (m)	
	Pure vertical load	Combination of loads	Pure vertical load	Combination of loads	Pure lateral load	Combination of loads
1	0.034	0.022	0.071	0.072	0.103	0.042
2	0.006	0.013	0.032	0.039	0.071	0.043
3	0.002	0.016	0.017	0.026	0.056	0.044
4	0.025	0.017	0.041	0.041	0.054	0.040
5	0.012	0.006	0.028	0.029	0.097	0.068
6	0.013	0.002	0.045	0.046	0.059	0.031
7	0.041	0.037	0.053	0.055	0.064	0.051
8	0.025	0.015	0.050	0.051	0.045	0.028
9	0.021	0.012	0.047	0.048	0.083	0.038

of squares and S_T is the total sum of squares. For a set of results, Y_1, Y_2... and Y_N, the total sum of squares can be calculated as:

$$S_T = \sum_{i=1}^{N} Y_i^2 - \frac{T^2}{N}; \quad T = (Y_1 + Y_2 + ... + Y_N) \tag{3}$$

And the total variance is equal to:

$$S_A = \frac{A_1^2}{N_{A_1}} + \frac{A_2^2}{N_{A_2}} + \frac{A_3^2}{N_{A_3}} - C.F;\; C.F = \frac{T^2}{N} \quad (4)$$

where C.F. is the correction factor, N_{A1} is the total number of experiments in which level 1 of factor A is present, and A_1 is the total of results (Y_i) that include factor A_1.

The pure sum of squares also can be calculated as:

$$S'_A = S_A - (V_e \times f_A) \quad (5)$$

where Ve is the variance for the error term (obtained by calculating error sum of squares and dividing by error of freedom degrees) and f_A is the freedom degree of factor A. The contribution percent of each factor listed in Table 6.

As shown in Table 6, the raft thickness has the highest effect on the reduction of differential settlements. In other words, increasing the thickness of the piled raft would significantly lead to reduce the differential settlements. Anagnostopolous and Georgiadis [19] have demonstrated that the maximum bending moment in the raft and the load carried by the piles increases by increasing the raft thickness. The thickness of raft has more effect on the maximum settlement than on the differential settlement and its effect increases when the raft thickness is low (flexible raft).

Increasing the pile length leads to increase in the bearing capacity ratio of the piles and consequently, the settlement reduces. However, the results show that the effect of pile length on reducing the maximum settlement is much higher than on the differential settlement. Also, the

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arrangement is the most effective factor in reducing the differential

Table 6. Contribution of each factor to achieve the optimal design (%)

Objective function	Minimum value of differential settlement		Minimum value of maximum settlement		Minimum value of horizontal deformation	
factor	Pure vertical load	Combination of loads	Pure vertical load	Combination of loads	Pure lateral load	Combination of loads
Arrangement	29.09	32.93	12.54	15.65	7.48	7.4
Raft thickness	62.2	54.23	34.61	33.88	2.82	11.3
Length	5.87	9.72	40.58	39.49	0.63	61.04
Diameter	2.84	3.12	12.27	10.98	89.07	20.26

settlement after the thickness of pile. This can be attributed to the type of loading. The loads applied to the piled raft by four columns and the load of each column is different. The C1 column has the maximum vertical load and thus the maximum settlement occurs around this column. When the pile length is increased, its bearing capacity increases and consequently, settlement around the C1 column is decreased. For evaluating the differential settlements, in addition to the maximum settlement, settlement in other parts is also important. The maximum settlement decreases due to the centralization of piles around the columns with maximum load in the concentrated pile arrangement. In this type of configuration, the number of piles around the other columns is less; therefore, the effect of the arrangement factor is more salient than the effect of the length to reach the minimum differential settlement.

As mentioned in pervious section, the lateral support surface of the pile and the pile stiffness increase by increasing in the pile diameter. Thus, the diameter factor has a maximum contribution percent in reducing the horizontal deformation under pure lateral load. Another important factor is the pile arrangement. Researchers have demonstrated that the effect of the group interaction decreases by increasing in the pile spacing and thereby, horizontal deformation decreases. In this study, the spacing of the piles from each other in the uniform arrangement is more than the two other arrangements. Hence, the uniform arrangement is appropriate for the piled raft under lateral load.

As can be seen in Table 6, the effect of the pile length on the horizontal deformation of the piled raft under pure lateral load is negligible; however, the pile length has a considerable effect in reducing the horizontal deformation in case of combination of loads. This issue was completely investigated in pervious section. The presence of vertical loads before the application of the lateral loads leads to an increase in the horizontal deformations. The reason for reduction in the horizontal deformation is an increase in the horizontal stress at the upper portion of the piles caused by relative displacement of the piles and their surrounding soil. On the other hand, this type of relative displacement increases by a decrease in the pile length. So, it is expected that the horizontal soil stresses at the upper portion of the subsurface increase by a decrease in the pile length. As a result, the less pile length leading the less horizontal deformations under load combinations become.

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Conclusion

The effects of difference factors on the horizontal deformation of the piled raft under pure lateral load and combination of loads have been investigated in this study. In all of the cases, due to the relative displacement between the pile and soil, the presence of vertical loads before lateral loads leads to an increase in the lateral resistance of the piled raft. Then, quantitatively comparison was conducted using the Taguchi method and the contribution ratio of each factor in reaching the optimal answer was determined. The results of these investigations are given as follows:

- The uniform arrangement with the pile spacing of 5m and two concentrated arrangements with pile spacing of 2.4 m and 3.5 m were compared. The results show that increasing the pile spacing leads to a decrease in the horizontal deformation of the piled raft under pure lateral load. The reason is that the interaction effect among the piles and the overlapping of failure zones (shadowing) is reduced. When the lateral load applied before the vertical load, the effect of the pile spacing became less noticeable.
- Comparison of the horizontal deformation of the piled rafts with three different pile diameters indicates that increasing the pile diameter leads to an increase in the lateral resistance of the piled raft. The reason is that increasing the pile diameter leads to an increase in the lateral support surface of the pile and hence horizontal deformation are reduced.

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- The thickness of the raft is one of the most effective factors in reducing the maximum and differential settlements. Such that, the settlements are considerably reduced by an increase in the raft thickness. In general, the presence of the raft increases the lateral resistance of the piled raft. In addition, the effect of raft thickness on the horizontal deformation increases by increasing the rigidity of the raft.
- When the piled raft is subjected to the lateral load, a variation of the pile length has no effect on the lateral resistance. As a result, if the lateral load is applied to the piled raft after the horizontal load, the pile length has an important role in increasing the lateral resistance. In a way that the lateral resistance increases by a decrease in the pile length. It should be noted that by reducing the pile length the relative displacement between the pile and soil increases, therefore, the horizontal soil stresses increase around the pile.

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